

## Function Operations

In business, the term *profit* is used to describe the difference between the money the business earns (revenue) and the money the business spends (cost).

A. Grooming USA charges \$25 for every pet that is groomed. Let  $x$  represent the number of pets groomed in a month. Define a revenue function for the business.

Enter your answer.



B. Materials and labor for each pet groomed cost \$15. The business also has fixed costs of \$1,000 each month. Define a cost function for this business.

Enter your answer.

C. Last month, Grooming USA groomed 95 pets. Did they earn a profit? What would the profit be if the business groomed 110 pets in a month?

Enter your answer.

D. **Generalize** Explain your procedure for calculating the profit for Grooming USA. Suppose you wanted to calculate the profit for several different scenarios. How could you simplify your process?

Enter your answer.



**Examples:** Add or subtract the following functions.

1. Given  $f(x) = 3x + 4$  and  $g(x) = x^2 - 5x + 2$ , find  $f + g$  and its domain.

2. Given  $f(x) = 3x + 4$  and  $g(x) = x^2 - 5x + 2$ , find  $f - g$  and its domain.

3. Given  $f(x) = 2x^2 + 7x - 1$  and  $g(x) = 3 - 2x$ , find  $f + g$  and its domain.

4. Given  $f(x) = 2x^2 + 7x - 1$  and  $g(x) = 3 - 2x$ , find  $f - g$  and its domain.

Examples: Multiply the following functions and find the domain of the resulting functions.

5.

The demand  $d$ , in units sold, for a company's new brand of cell phone at price  $x$ , in dollars, is  $d(x) = 5,000 - 10x$ . What is the company's expected revenue from cell phone sales in terms of the price,  $x$ ?



6. Suppose demand,  $d$ , for a company's product at cost,  $x$ , is predicted by the function  $d(x) = -0.25x^2 + 1000$ , and the price,  $p$ , that the company can charge for the product is given by  $p(x) = x + 16$ . Find the company's revenue function.

Examples: Divide the following functions.

7. Given  $f(x) = x - 7$  and  $g(x) = 2x^2 - 13x - 7$ , find  $\frac{f}{g}$  and its domain.

8. Given  $f(x) = x - 3$  and  $g(x) = x^2 - x - 6$ , find  $\frac{f}{g}$  and its domain.

9. Given  $f(x) = x^2 - 3x - 18$  and  $g(x) = x + 3$ , find  $\frac{f}{g}$  and its domain.

Examples: Find the following compositions of functions.

10. Given  $f(x) = x^2$  and  $g(x) = x + 1$ , find  $f(g(3))$ .

11. Given  $f(x) = 2x - 1$  and  $g(x) = 3x$ , find  $f(g(2))$ .

Examples: Find the following compositions of functions and the domain of the resulting functions.

12. Given  $f(x) = x^2$  and  $g(x) = x + 1$ , find  $f(g(x))$ .

13. Given  $f(x) = 2x - 1$  and  $g(x) = 3x$ , find  $f(g(x))$ .

14. Given  $f(x) = \sqrt{x+7}$  and  $g(x) = 2x - 5$ , find  $(f \circ g)(x)$ .

15. Given  $f(x) = x^2 + x + 2$  and  $g(x) = 4 - x$ , find  $(f \circ g)(x)$ .

16. Given  $f(x) = x^2 + 1$  and  $g(x) = x - 5$ , find  $(f \circ g)(x)$ .

### CONCEPT SUMMARY Function Operations

#### Add or Subtract Functions

**ALGEBRA**

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

**WORDS** The domain of the sum or difference of  $f$  and  $g$  is the intersection of the domain of  $f$  and the domain of  $g$ .

**NUMBERS** For  $f(x) = 3x + 5$  and  $g(x) = x - 3$ ,  $f + g = (3x + 5) + (x - 3) = 4x + 2$  and  $f - g = (3x + 5) - (x - 3) = 2x + 8$

#### Multiply or Divide Functions

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

The domain is the set of all real numbers for which  $f$  and  $g$  and the new function are defined.

For  $f(x) = 3x + 5$  and  $g(x) = x - 3$ ,  $f \cdot g = (3x + 5)(x - 3) = 3x^2 - 4x - 15$  and  $\frac{f}{g} = \frac{3x+5}{x-3}$  for  $x \neq 3$

#### Compose Functions

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

The domain of  $f \circ g$  is the set of all real numbers  $x$ , in the domain of  $g$ , such that  $g(x)$  is in the domain of  $f$ .

For  $f(x) = 3x + 5$  and  $g(x) = x - 3$ ,  $f \circ g = 3(x - 3) + 5 = 3x - 4$  and  $g \circ f = (3x + 5) - 3 = 3x + 2$